

also some remarks as to the mode of calculation of the new tables, and also as to a peculiar symmetry of the numbers in the tables of each set, a symmetry which, so far as I am aware, has not hitherto been observed, and the existence of which appears to constitute an important theorem in the subject. The theorem in question might, I think, be deduced from a very elegant formula of M. Borchardt (referred to in the sequel), which gives the generating function of any symmetric function of the roots, and contains potentially a method for the calculation of the tables (*b*), but which, from the example I have given, would not appear to be a very convenient one for actual calculation.

VI. "Memoir on the Conditions for the Existence of given Systems of Equalities among the Roots of an Equation."

By ARTHUR CAYLEY, Esq., F.R.S. Received December 18, 1856.

(Abstract.)

It is well known that there is a symmetric function of the roots of an equation, viz. the product of the squares of the differences of the roots, which vanishes when any two roots are put equal to each other, and that consequently such function expressed in terms of the coefficients and equated to zero, gives the condition for the existence of a pair of equal roots. And it was remarked long ago by Professor Sylvester, in some of his earlier papers in the 'Philosophical Magazine,' that the like method could be applied to finding the conditions for the existence of other systems of equalities among the roots, viz. that it was possible to form symmetric functions, each of them a sum of terms containing the product of a certain number of the differences of the roots, and such that the entire function might vanish for the particular system of equalities in question; and that such functions expressed in terms of the coefficients and equated to zero would give the required conditions. The object of the present memoir is to extend this theory, and render it exhaustive by showing how to form a series of types of all the different functions which vanish for one or more systems of equalities among the roots; and in particular to obtain by the method distinctive conditions for all the different

systems of equalities between the roots of a quartic or a quintic equation, viz. for each system conditions which are satisfied for the particular system, and are not satisfied for any other systems, except, of course, the more special systems included in the particular system. The question of finding the conditions for any particular system of equalities is essentially an indeterminate one, for given any set of functions which vanish, a function syzygetically connected with these will also vanish; the discussion of the nature of the syzygetic relations between the different functions which vanish for any particular system of equalities, and of the order of the system composed of the several conditions for the particular system of equalities, does not enter into the plan of the present memoir. I have referred here to the indeterminateness of the question for the sake of the remark that I have availed myself thereof, to express by means of invariants or covariants the different systems of conditions obtained in the sequel of the memoir; the expressions of the different invariants and covariants referred to are given in my "Second Memoir upon Quantics," Phil. Trans. vol. cxlvi. (1856).

VII. "Tables of the Sturmian Functions for Equations of the Second, Third, Fourth and Fifth Degrees." By ARTHUR CAYLEY, Esq., F.R.S. Received December 18, 1856.

(Abstract.)

The general expressions for the Sturmian functions in the form of determinants, are at once deducible from the researches of Professor Sylvester in his early papers on the subject in the 'Philosophical Magazine,' and in giving these expressions in the memoir "Nouvelles Recherches sur les Fonctions de M. Sturm," Liouville, t. xiii. p. 269 (1848), I was wrong in claiming for them any novelty. The expressions in the last-mentioned memoir admit of a modification by which their form is rendered somewhat more elegant; I propose, on the present occasion, merely to give this modified form of the general expression, and to give the developed expressions of the functions in question for equations of the degrees, two, three, four and five.